Normal Distribution

What is a normal distribution?

A normal distribution is a bell-shaped curve that describes how data are spread around a mean (average).

In a normal distribution:

- most values are close to the mean
- fewer values occur further from the mean
- the curve is symmetrical about the mean

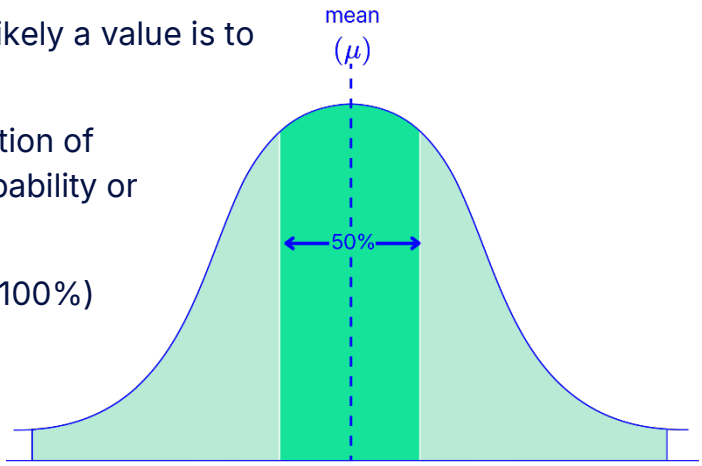
Why is the normal distribution useful?

A normal distribution helps us understand how likely a value is to occur.

The area under the curve represents the proportion of observations, which can be expressed as a probability or percentage.

The total area under the curve always equals 1 (100%)

- Area of 0.50 = 50% of observations
- Area of 0.25 = 25% of observations
- Area of 0.95 = 95% of observations



Therefore:

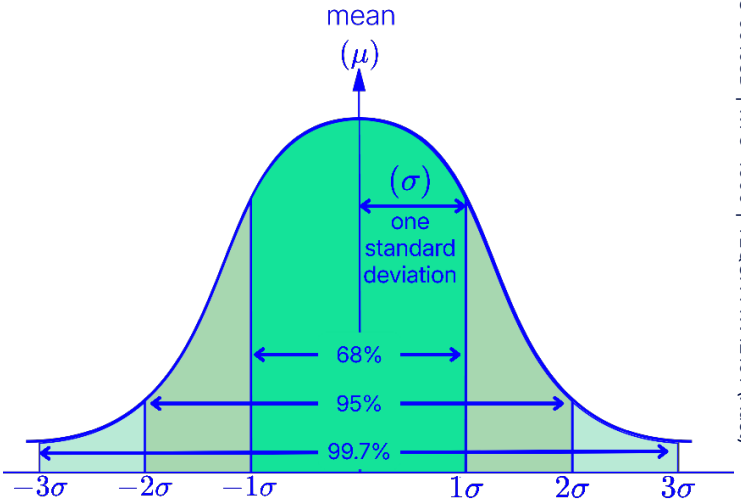
Area under the curve = Percentage of the population = Probability of an observation occurring

The Empirical Rule

For normal distributions, observations tend to cluster around the mean.

The Empirical Rule gives the approximate proportion of observations within 1, 2 and 3 standard deviations of the mean:

Range	Percentage	Probability
$\mu \pm 1\sigma$	68%	0.68
$\mu \pm 2\sigma$	95%	0.95
$\mu \pm 3\sigma$	99.7%	0.997



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Using z-scores to find probability

The Empirical Rule only gives probabilities for common intervals. To calculate probabilities, we first need to know **where a value lies on the normal distribution**.

A z-score tells us how far a value is located on the normal distribution using the formula below:

$$z = \frac{x - \mu}{\sigma}$$

where:
x = observed value
μ = mean
σ = standard deviation

Once a z-score has been calculated, we use a **standard normal table (z-table)** to find the area under the curve.

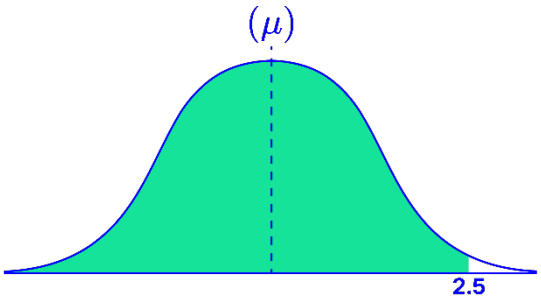
Common probability questions

Less than a value

$$P(Z < 2.5)$$

$$= 0.9938$$

99.38% of data lie below $z = 2.5$



Greater than a value

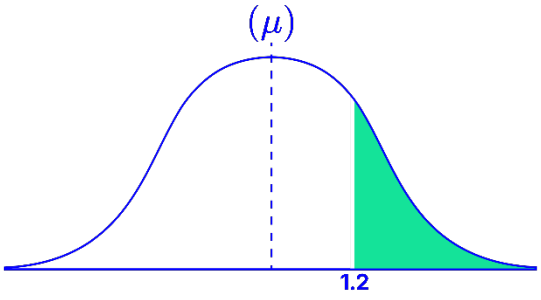
$$P(Z > 1.2)$$

$$= 1 - P(Z < 1.2)$$

$$P(Z < 1.2) = 0.8849$$

$$= 1 - 0.8849 = 0.1151$$

11.51% of data lie above $z = 1.2$



Between two values

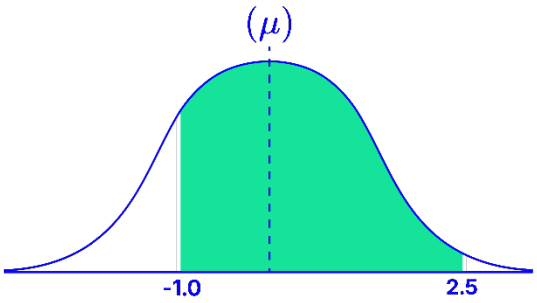
$$P(-1.0 < Z < 2.5)$$

$$= P(Z < 2.5) - P(Z < -1)$$

$$= 0.9938 - 0.1587$$

$$= 0.8351$$

83.51% of data lie between $z = -1$ and 2.5



Related helpsheets

- Calculating Z-Scores
- Normality in Jamovi

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