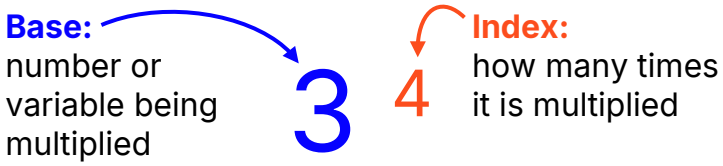
Index Laws

What is an index?

An index (or exponent) tells us how many times a base is multiplied by itself (a power).



Example

Index form	Expanded form	Evaluated
3^4	$3 \times 3 \times 3 \times 3$	$= 81$
5^3	$5 \times 5 \times 5$	$= 125$
$a^2 \times b^3$	$a \times a \times b \times b \times b$	

Multiplying powers

When multiplying terms with the **same base**, **add** the indices.

$$3^2 \times 3^3$$
$$= 3^{2+3}$$
$$= 3^5$$

$$\underbrace{3 \times 3 \quad \times \quad 3 \times 3 \times 3}_{5}$$

∴ the generic formula = $a^m \times a^n = a^{m+n}$

Dividing powers

When dividing terms with the **same base**, **subtract** the indices.

$$3^5 \div 3^3$$
$$= 3^{5-3}$$
$$= 3^2$$

$$\frac{3 \times 3 \times \cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3}}$$
$$= 3 \times 3$$

∴ the generic formula = $a^m \div a^n = a^{m-n}$

Zero power

Any non-zero number raised to the power of zero equals 1.

$$\frac{2}{2} = 1 \therefore = \frac{2^1}{2^1}$$

(Applying division index law)

$$= 2^{1-1}$$
$$= 2^0 = 1$$

∴ the generic formula = $a^0 = 1$

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Power of a power

When a power is raised to another power, multiply the indices.

$$(3^2)^3 = 3^{2\times3} = 3^6$$

$$3^2 \times 3^2 \times 3^2 = 3^{2+2+2}$$

∴ the generic formula = $(a^m)^n = a^{m\times n}$

Power of a product

Raise every factor inside the brackets to the power outside.

$$(2a)^3$$

$$2a \times 2a \times 2a = 2^3 \times a^3 = 8a^3$$

∴ the generic formula = $(ab)^n = a^n b^n$

Power of a quotient

Raise every factor inside the brackets to the power outside.

$$\left(\frac{2}{a}\right)^3$$

$$\frac{2}{a} \times \frac{2}{a} \times \frac{2}{a} = \frac{2^3}{a^3} = \frac{8}{a^3}$$

∴ the generic formula = $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Negative index

A negative index does **not** make a value negative.
It tells us to write the term as a fraction.

$$3^2 \div 3^3 = \frac{\cancel{3} \times \cancel{3}}{3 \times \cancel{3} \times \cancel{3}} = \frac{1}{3}$$

(Applying division index law)

$$3^{2-3} = 3^{-1}$$

$$\therefore 3^{-1} = \frac{1}{3}$$

∴ the generic formula = $a^{-n} = \frac{1}{a^n}$

Related helpsheets

- Scientific Notation
- Algebraic Expression

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